PG Comparison & Other Thoughts

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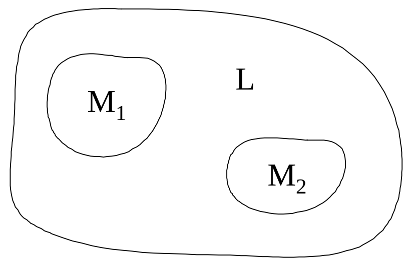
Motivation

- **How do you compare PG's to each other?**
  - Equivalent Damage
    - Stress Life: Good?
    - Strain Life: Good
- **How do you compare PG's to accelerated tests?**
  - Equivalent Damage
    - Stress Life: Poor
    - Strain Life: Good

The Component

- **Suppose a component exists with:**
  - a nominally elastic region, L
  - one or more non-nominally elastic regions, M_i



Neuber's Rule

This is an equation to convert elastic stress and strain into true stress and true strain:

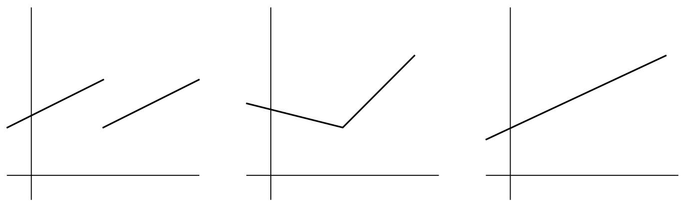
$$S \cdot e \cdot K_t^2 = \sigma \cdot \varepsilon$$

where S and e are elastic stress and elastic strain, respectively, K_t is the stress concentration factor, and σ and ε are the true stress and true strain, respectively.

Continuity

Continuity:

- **If you can assume at least C^0 continuity, you can make certain assumptions**
  - One assumption is that the product of two continuous functions is also continuous



Discontinuous | Continuous, C^0 | Continuous and first derivative continuous, C^1

"Load"-to-Strain Parameter

In a linearly elastic component, the elastic strain, e, at any location can be related to the applied "load", V, by a single parameter, β:

$$e = V\beta$$

Notes:

- β varies continuously across a component (at least C^0)
- β can have values from 0 to a very large number
- V is an input signal and could be load, displacement, strain, angle, voltage, or any other measurable parameter

Stress Concentration Factor, K_t

The stress at any location in an *M* region can be related to a location in an *L* region

$$\sigma = K_t S$$

Notes:

- K varies continuously across a component (at least C^0)
- K can have values from 1 to a very large number

Neuber's Equation

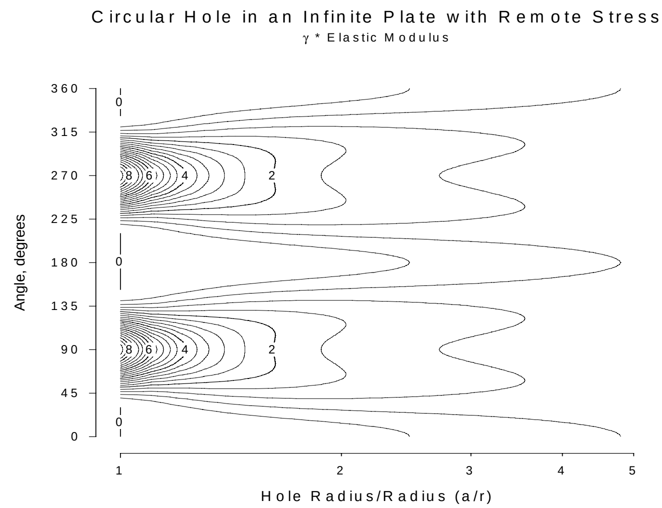
Substituting β in to Neuber's equation and rearranging:

$$V^2 \beta^2 E K_t^2 = \sigma\varepsilon$$

A new parameter γ can be defined:

$$\gamma = \beta^2 E K_t^2$$

Example of a γ field



Implications of γ

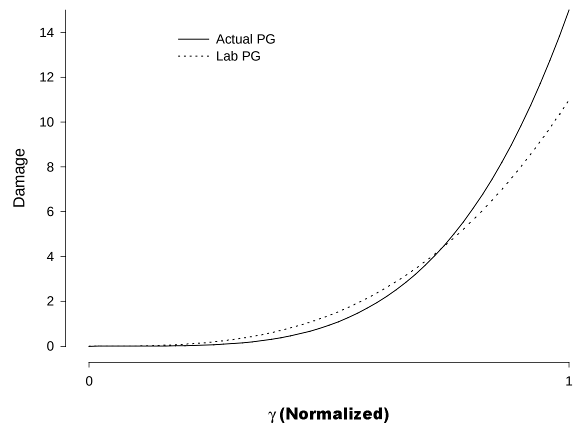
γ has the following properties:

- Varies continuously across the component (at least C^0)
- Has values from 0 to a very large number
- One value of γ corresponds to one value of fatigue life (or damage), for a specific material and loading spectra

This means that

- Fatigue life estimations can be performed for only a few values of γ and all other fatigue life estimations will lie on a curve drawn through these few values of γ
- A fatigue life estimate for a complex, unknown component can be performed using several values of γ, given a knowledge of β's and K 's

Result



Curve Comparison

Qualitatively

- **Bliss**
  - Both curves lie atop each other
- **Happiness:**
  - Curves cross at Damage = 1.0
- **Angst:**
  - Curves do not cross

Quantitatively

- **Challenge**
  - Comparison (difference) at each target gamma point?

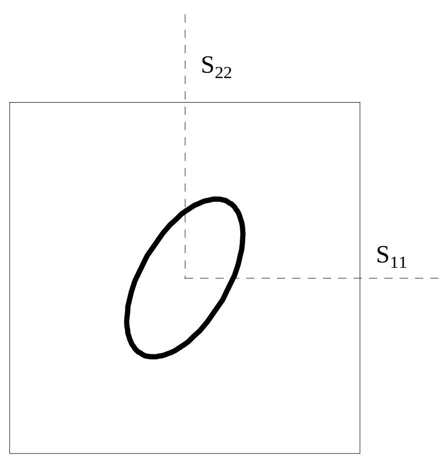
Extension: N independent loads?

- **Change notation:**
  - (Much hand-waving)
    - Similar principles apply
  - Model not defined, using:

$$[\varepsilon] = f([S],[V])$$

Failure Locus in 2D

- **A smooth shape**
- **Not necessarily an ellipse**
- **Full surface could be modeled reasonably using NURBS and a 5 x 5 net**



How does this help?

- The general shapes of the failure surfaces should be manifolds
- Using your "favorite" model, you can establish what the surface looks like using NURBS
- You can compare PG's or use it to design a part (this might be a bit more difficult)

Future

- Establish a general model for the failure surfaces that relate to the plane strain condition
- Extend to a large stamped steel structure, such as a vehicle frame
  - 100 independent loads
  - More than 6 loads need to be combined