SAE Fatigue Design & Evaluation Committee

Charlie Sieck

25 April 2013

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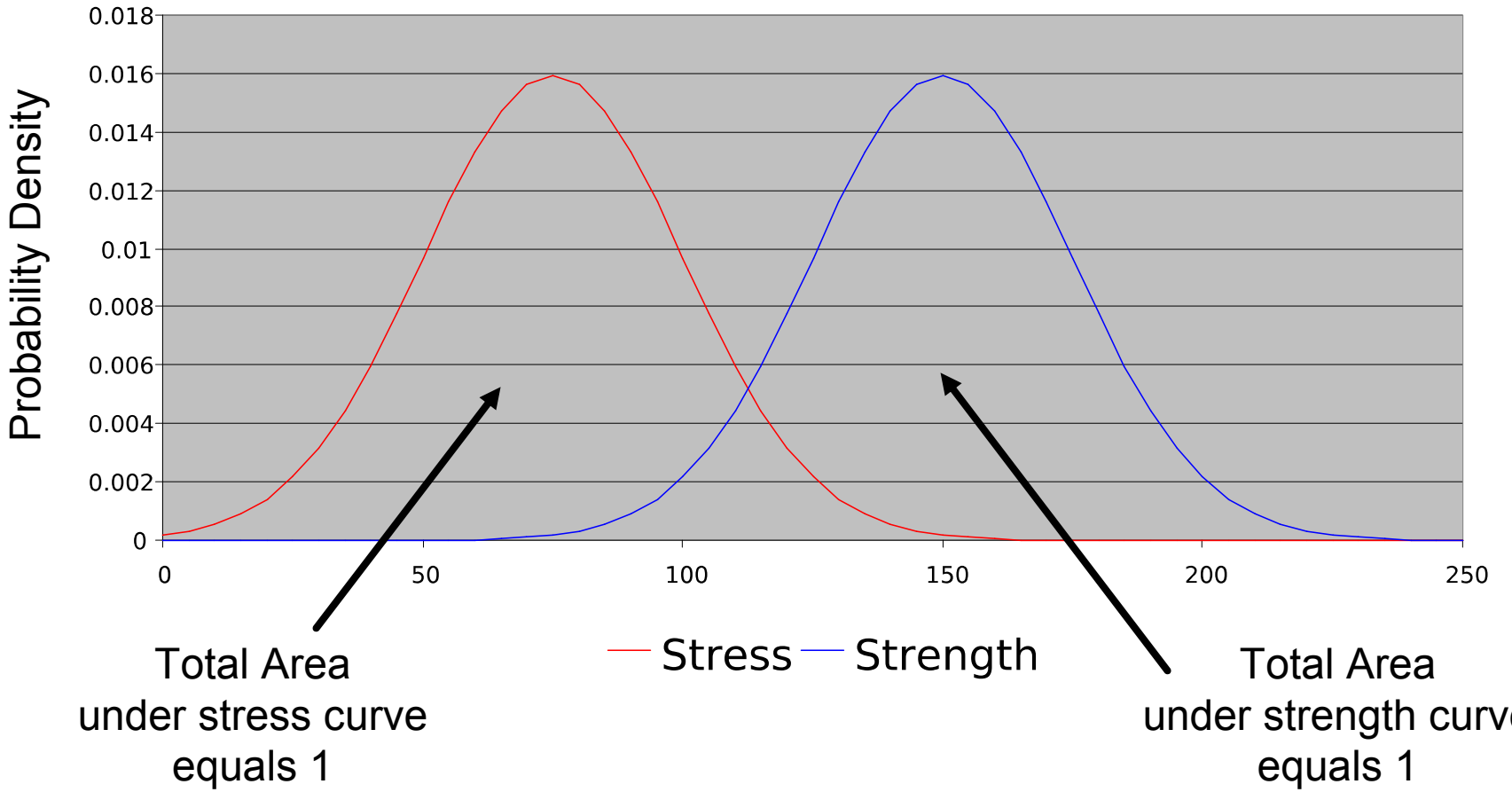
Basic Problem

Stress
Mean = 75
Stdev = 25

Strength
Mean = 150
Stdev = 25



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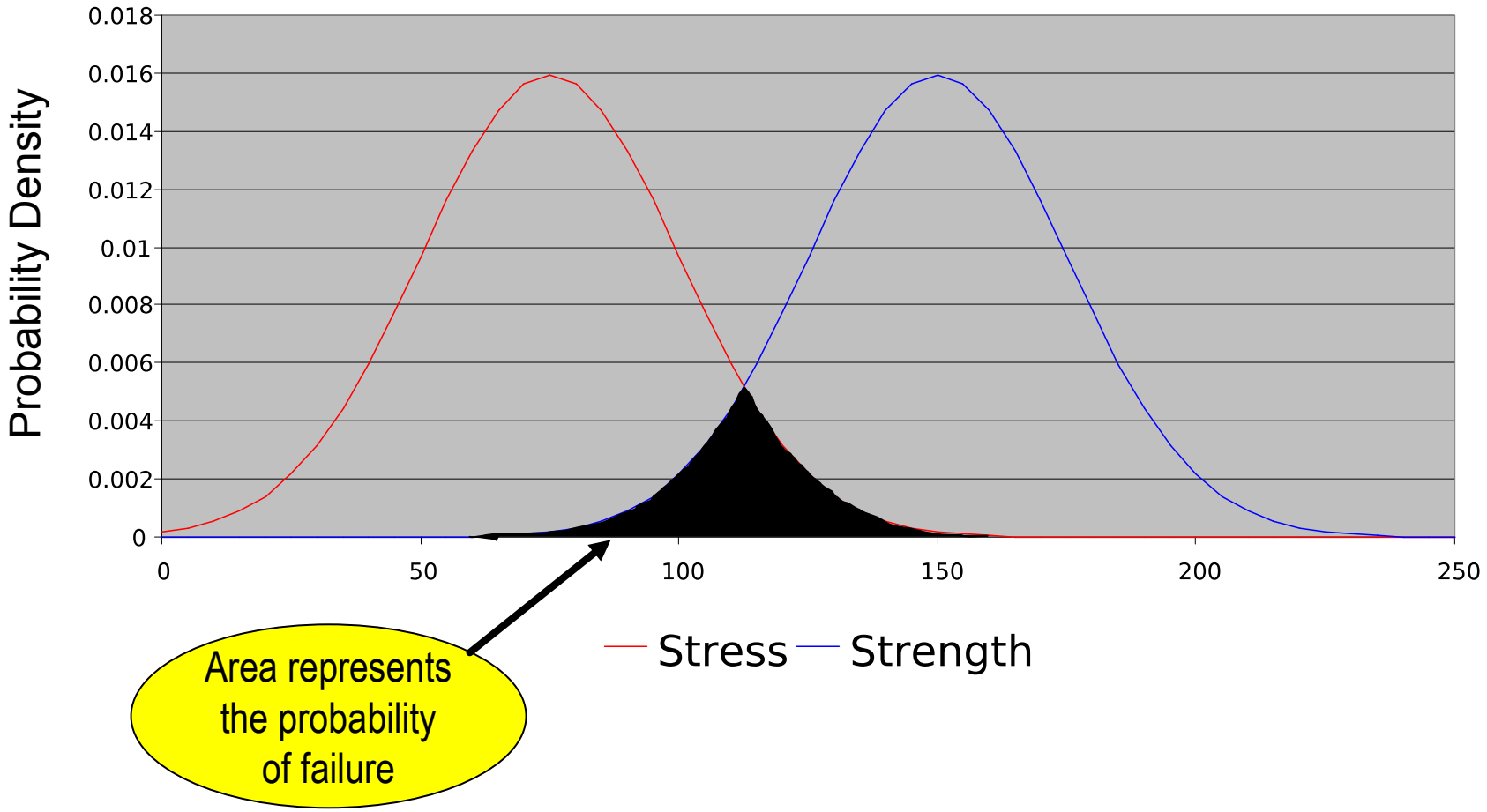
Misconception

Stress
Mean = 75
Stdev = 25

Strength
Mean = 150
Stdev = 25



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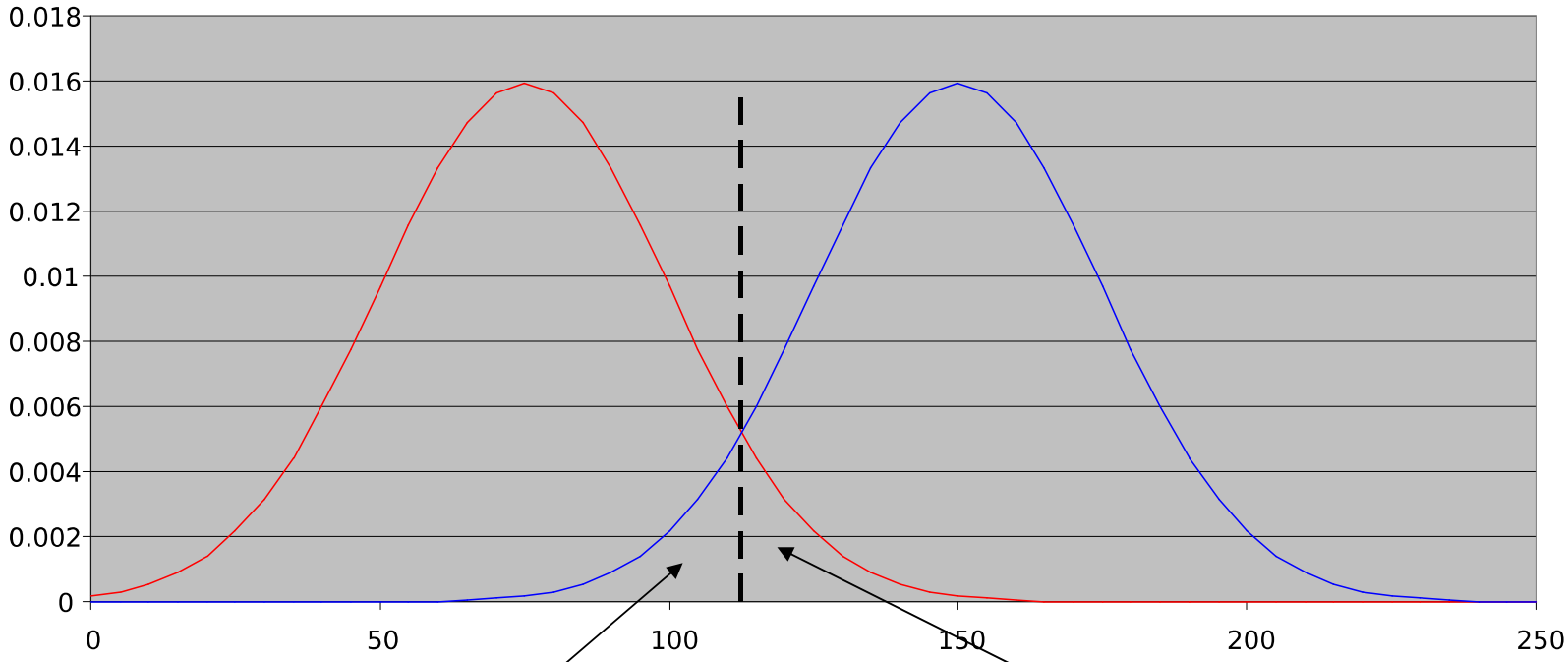
Calculating the Area

Stress
Mean = 75
Stdev = 25

Strength
Mean = 150
Stdev = 25



6.68% of Strength

6.68% of Stress

Total Area
13.36%?

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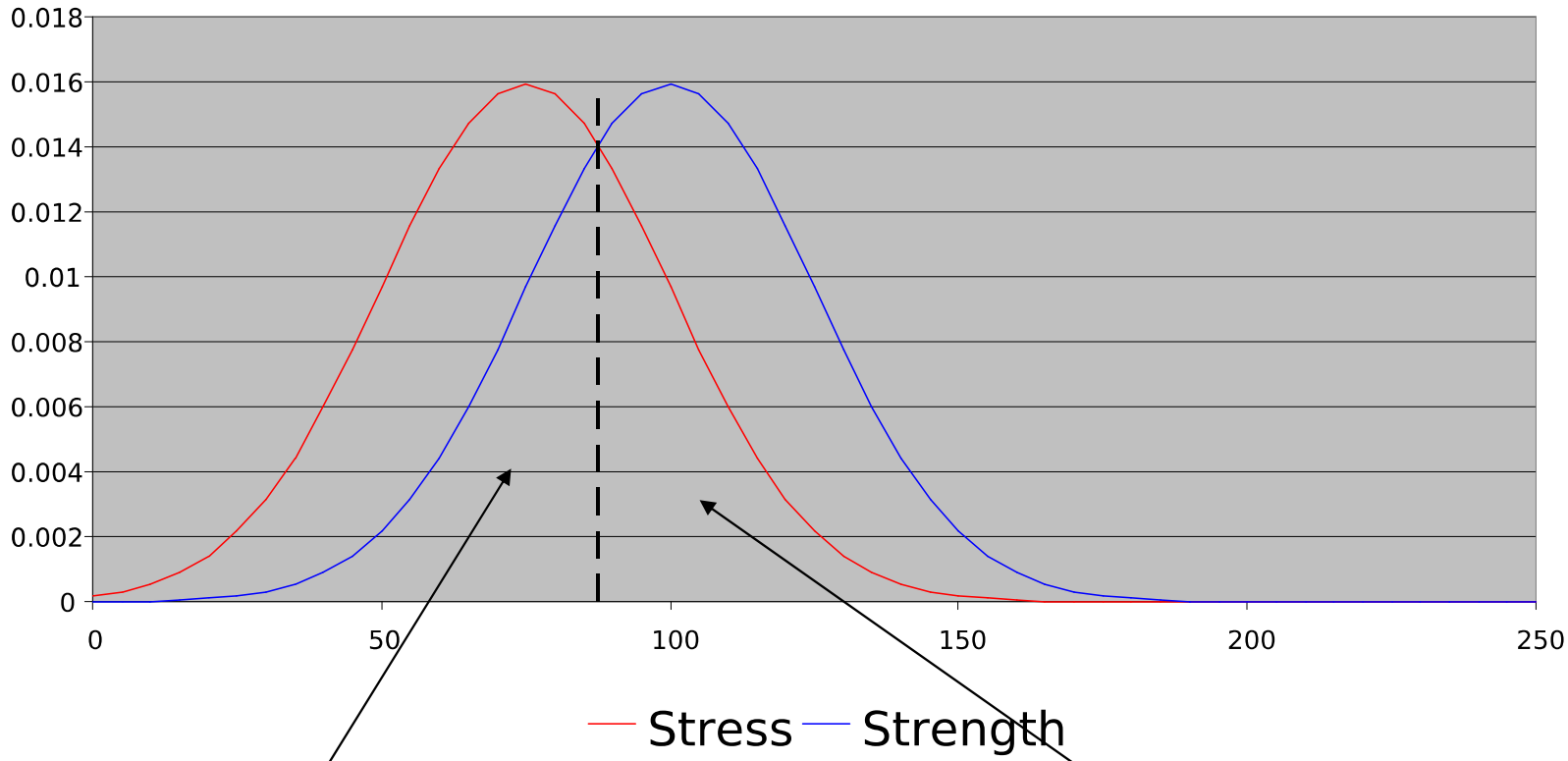
Stress
Mean = 75
Stdev = 25

Reducing Strength by 50

Strength
Mean = 100
Stdev = 25



30.85% of Strength

Total Area
61.7%?

30.85% of Stress

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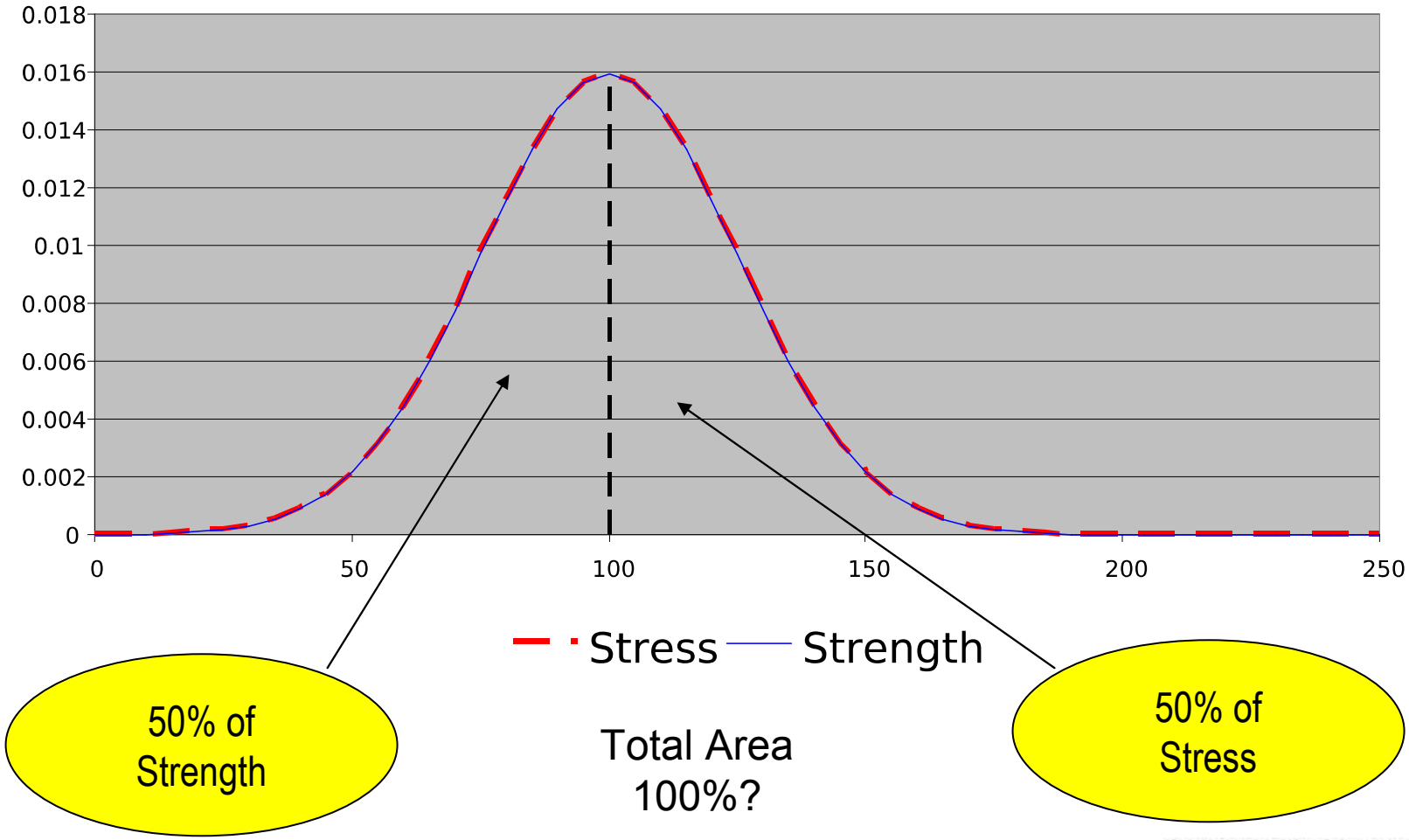
Stress
Mean = 75
Stdev = 25

Increasing Stress by 25

Strength
Mean = 150
Stdev = 25



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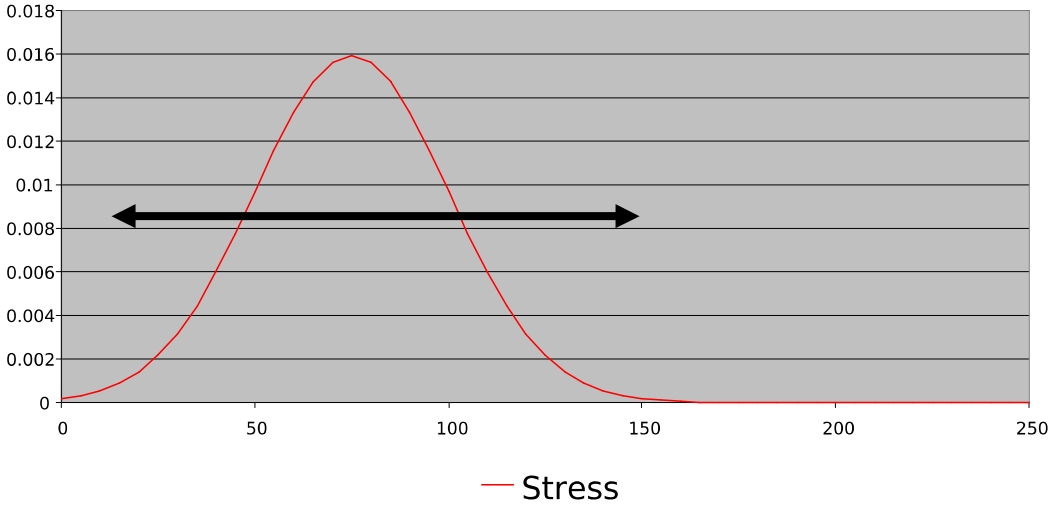
Correct Perspective

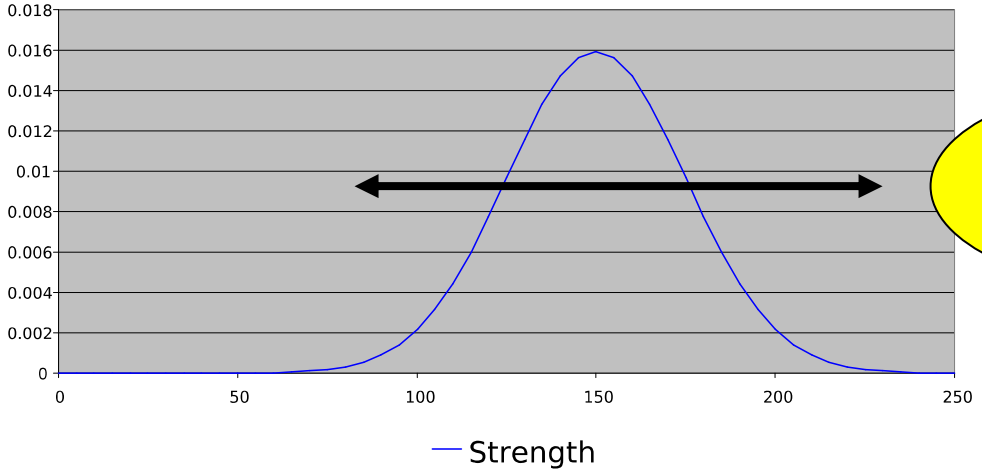
Stress
Mean = 75
Stdev = 25

Strength
Mean = 150
Stdev = 25



Stress
Can take on
Any values

Let's assume
Stress and
Strength
are Independent



Strength
Can take on
Any values

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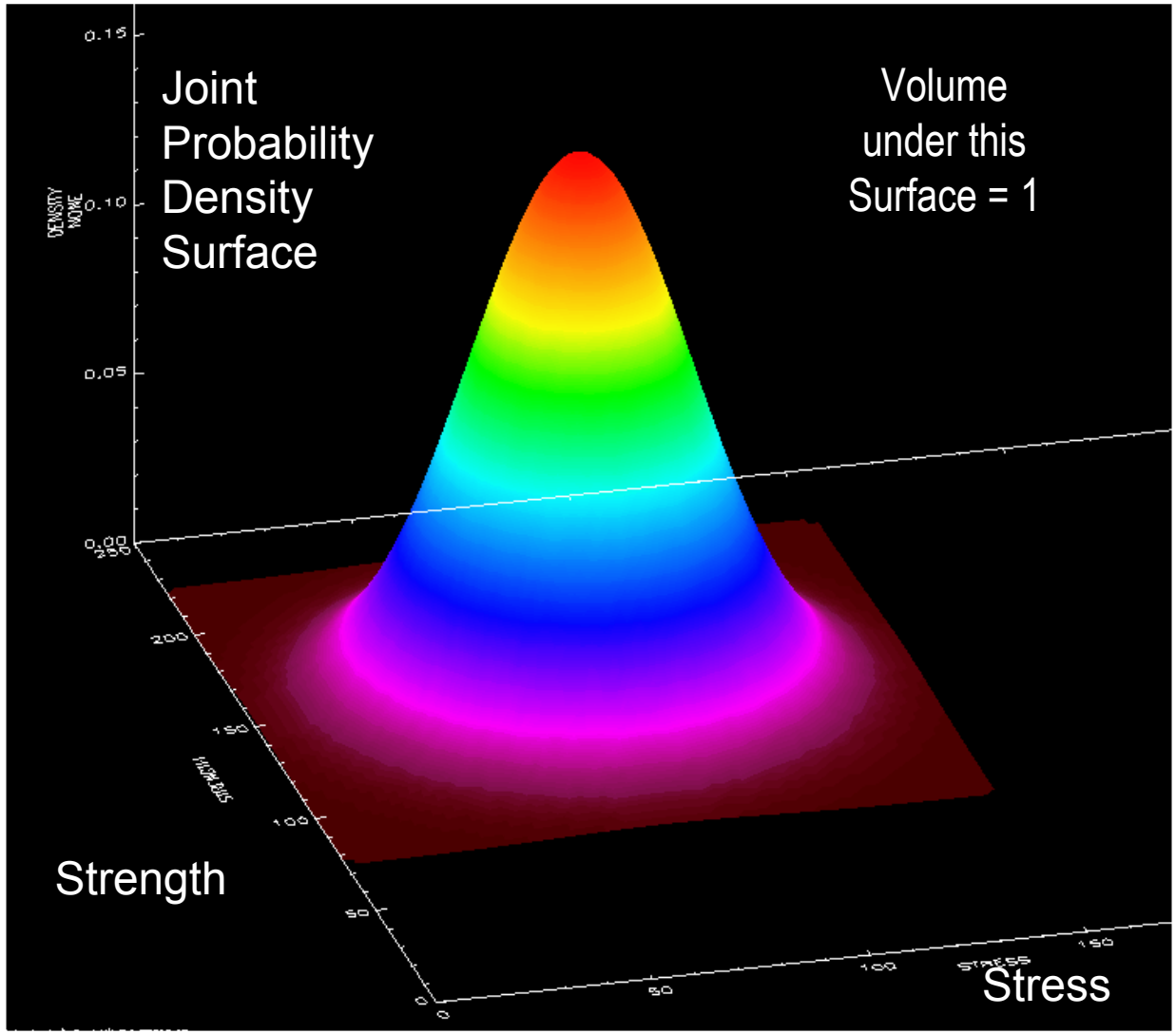
Stress
Mean = 75
Stdev = 25

3D Surface of Joint Probability Density between Stress and Strength

Strength
Mean = 150
Stdev = 25



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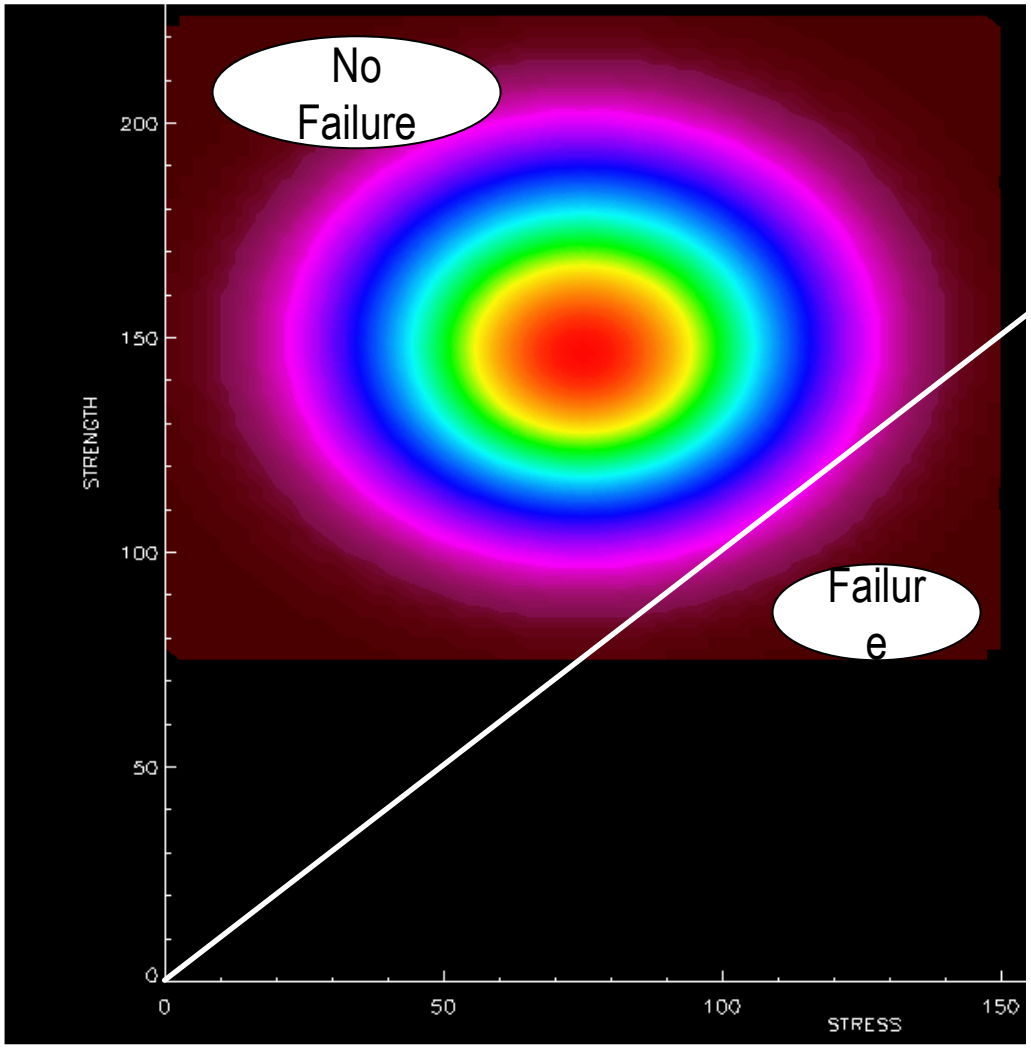
Top View of Joint Probability Density Surface

Stress
Mean = 75
Stdev = 25

Strength
Mean = 150
Stdev = 25



Strength

Stress

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Practical Application

If Stress has a normal statistical distribution

and

Strength has a normal statistical distribution,

then

(Strength – Stress)

will have a normal statistical distribution.

(Strength – Stress) = Performance Function

Failure will occur when Performance Function <= 0

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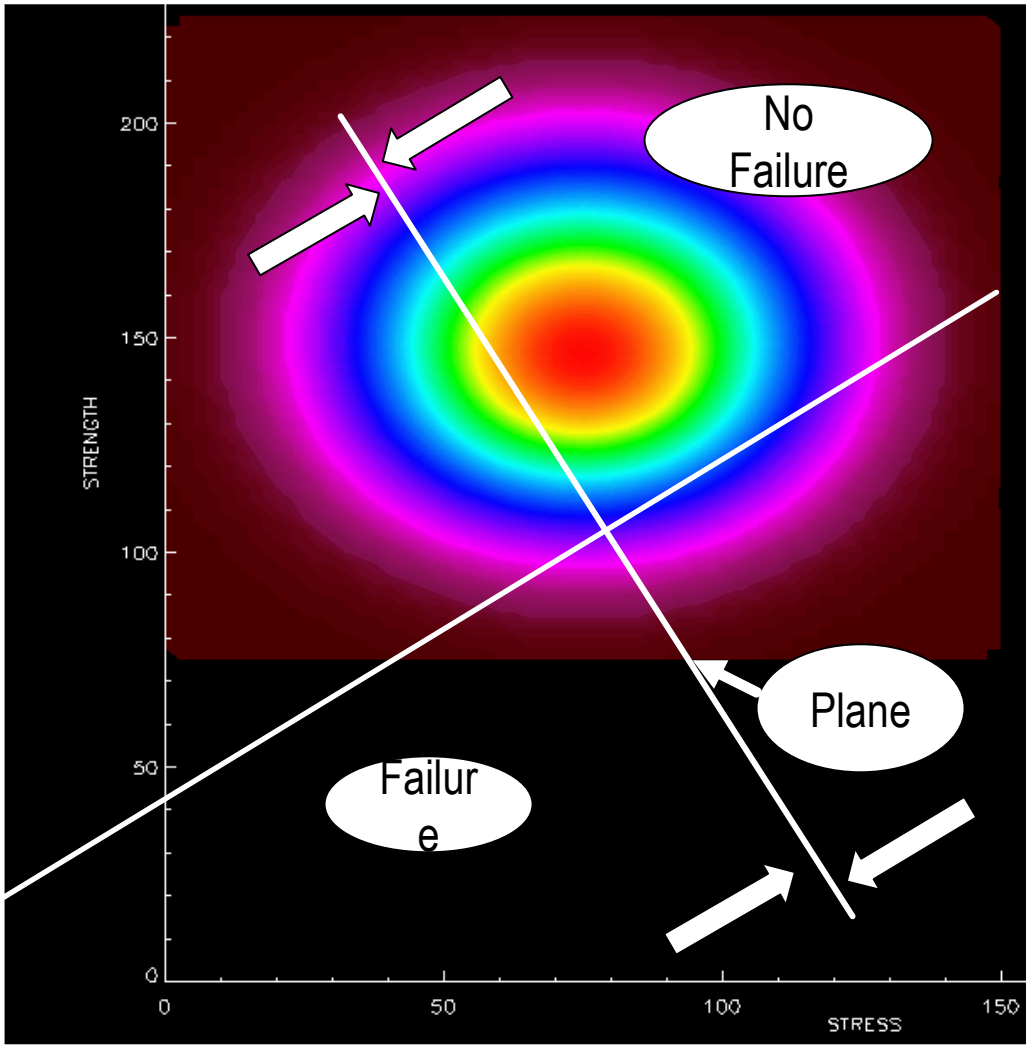
Top View of Surface

Stress
Mean = 75
Stdev = 25

Strength
Mean = 150
Stdev = 25



Stress < Strength

Stress = Strength

Stress > Strength

Strength

Stress

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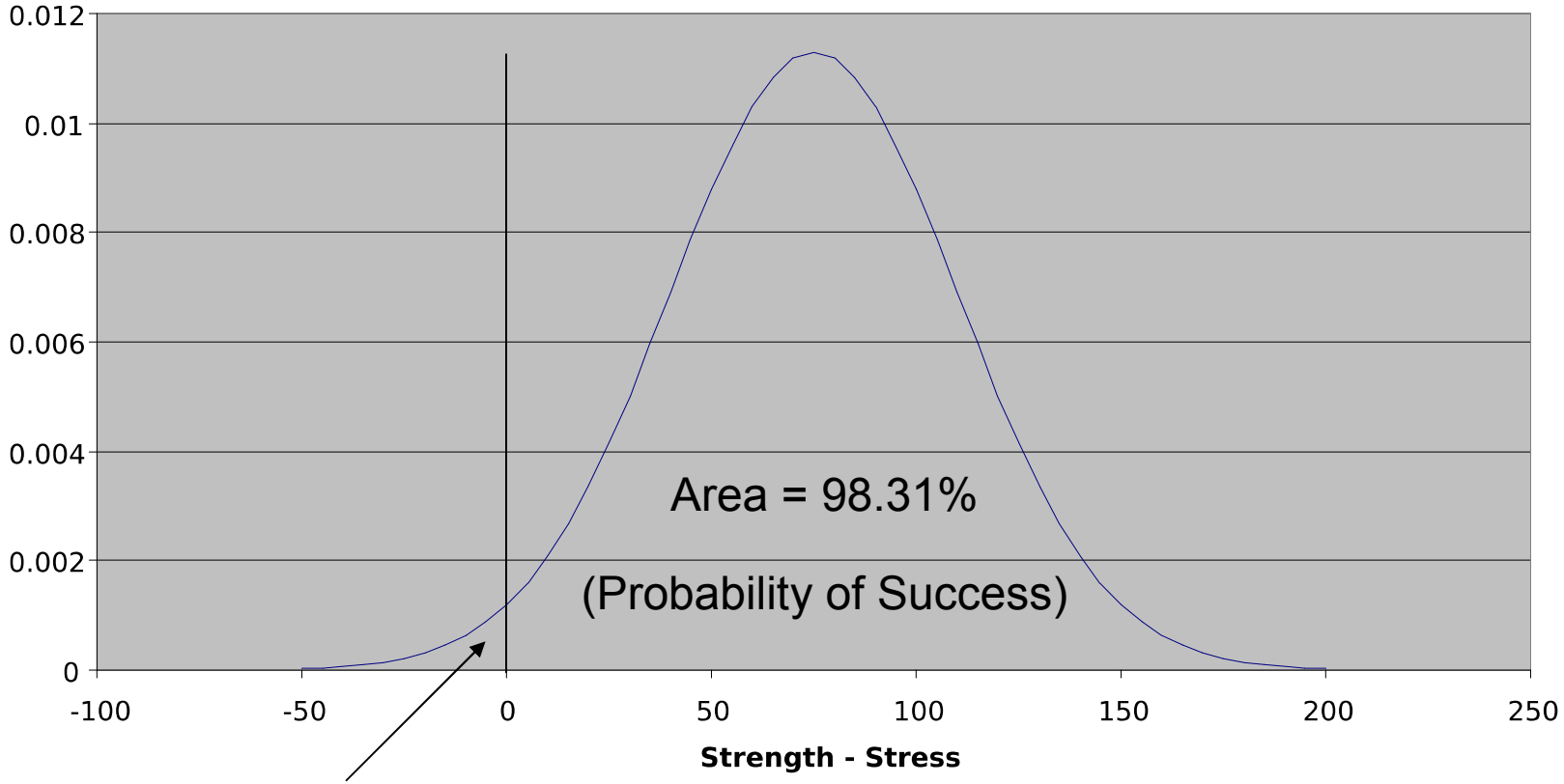
Stress
Mean = 75
Stdev = 25

Probability Density of (Strength – Stress)

Strength
Mean = 150
Stdev = 25



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Math

$$\text{Mean}_{\text{Strength}} - \text{Mean}_{\text{Stress}}$$

$$\text{Standard Deviation} = \sqrt{\text{Standard Deviation}_{\text{Strength}}^2 + \text{Standard Deviation}_{\text{Stress}}^2}$$

Comparison of Results

Stress		Strength		Wrong Overlap Area	Correct Failure Rate
Mean	Stdev	Mean	Stdev		
75	25	150	25	13.36%	1.69%
75	25	100	25	61.70%	23.98%
100	25	100	25	100%	50.0%

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Bathtub Curve

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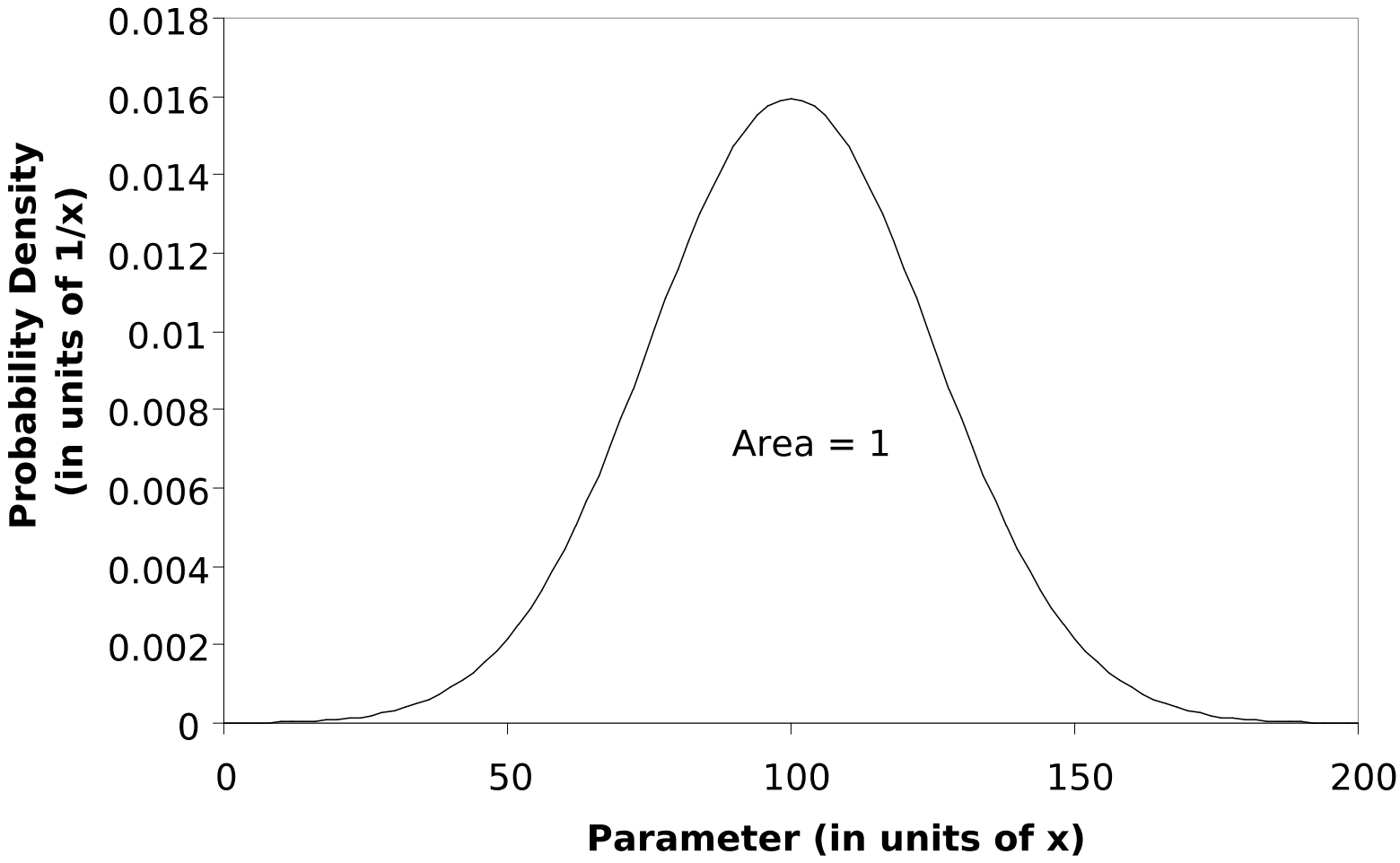
Probability Density Function (PDF)



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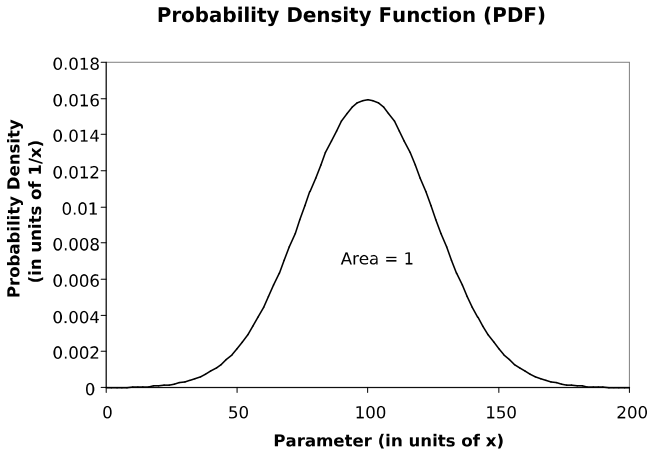
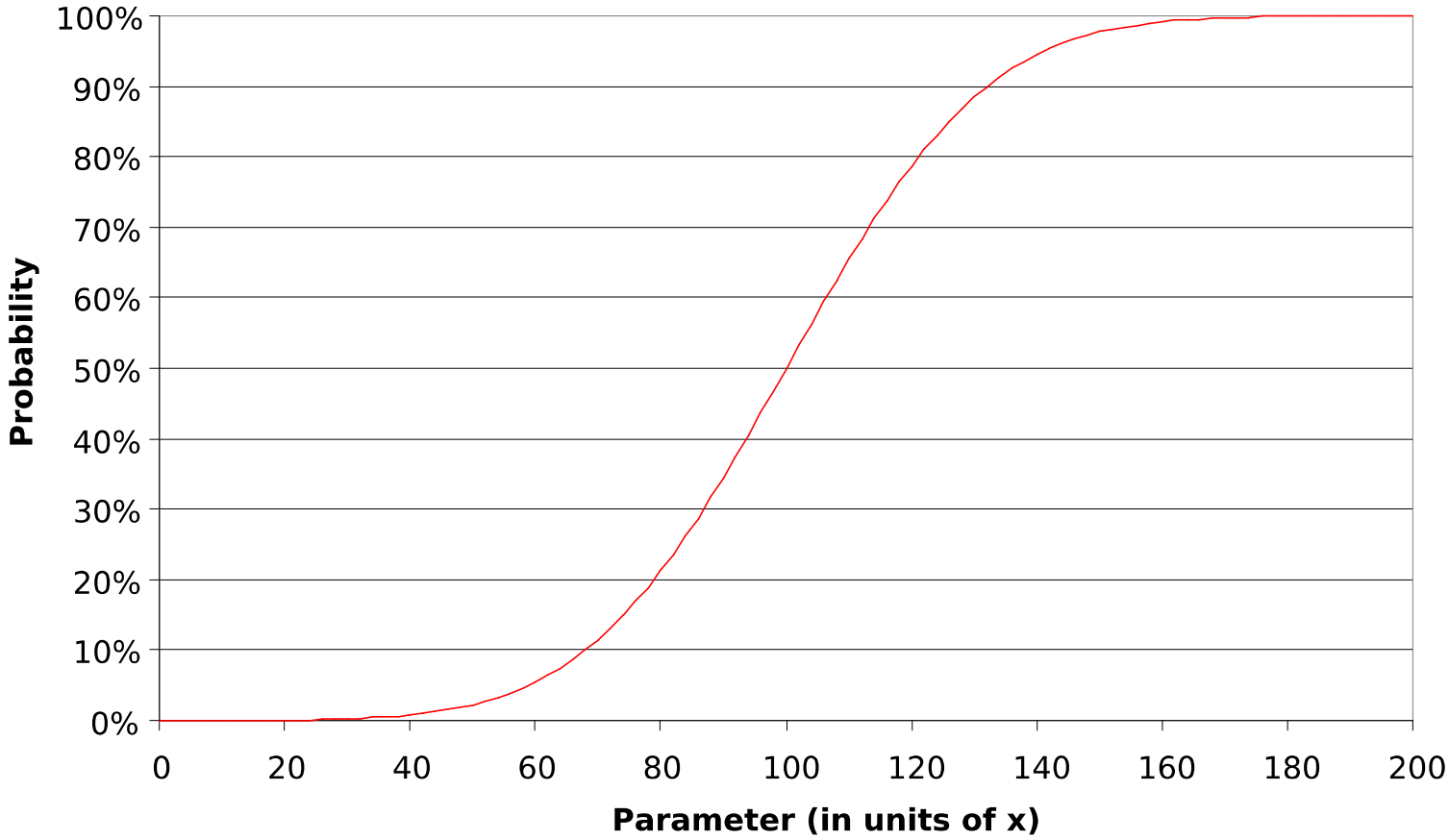
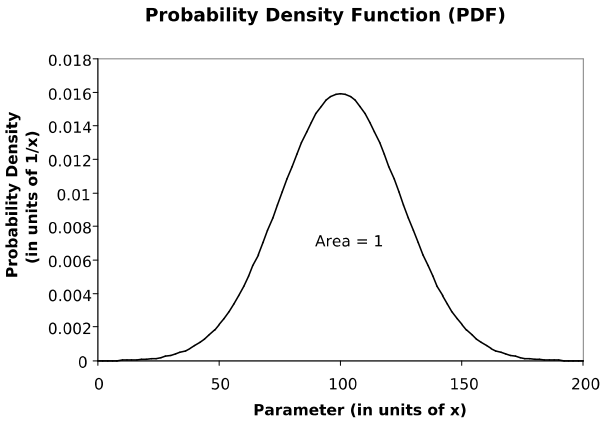


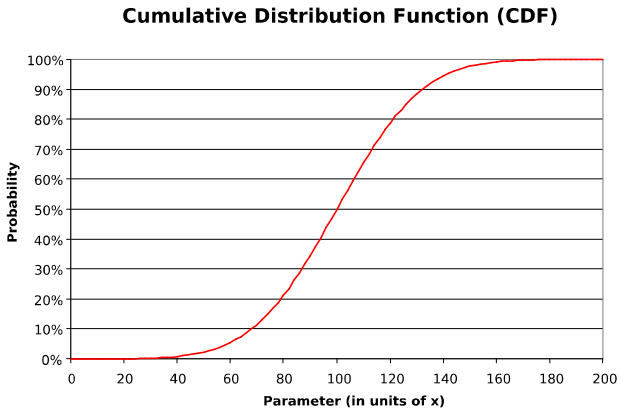
Integrating the PDF creates the CDF

Cumulative Distribution Function (CDF)

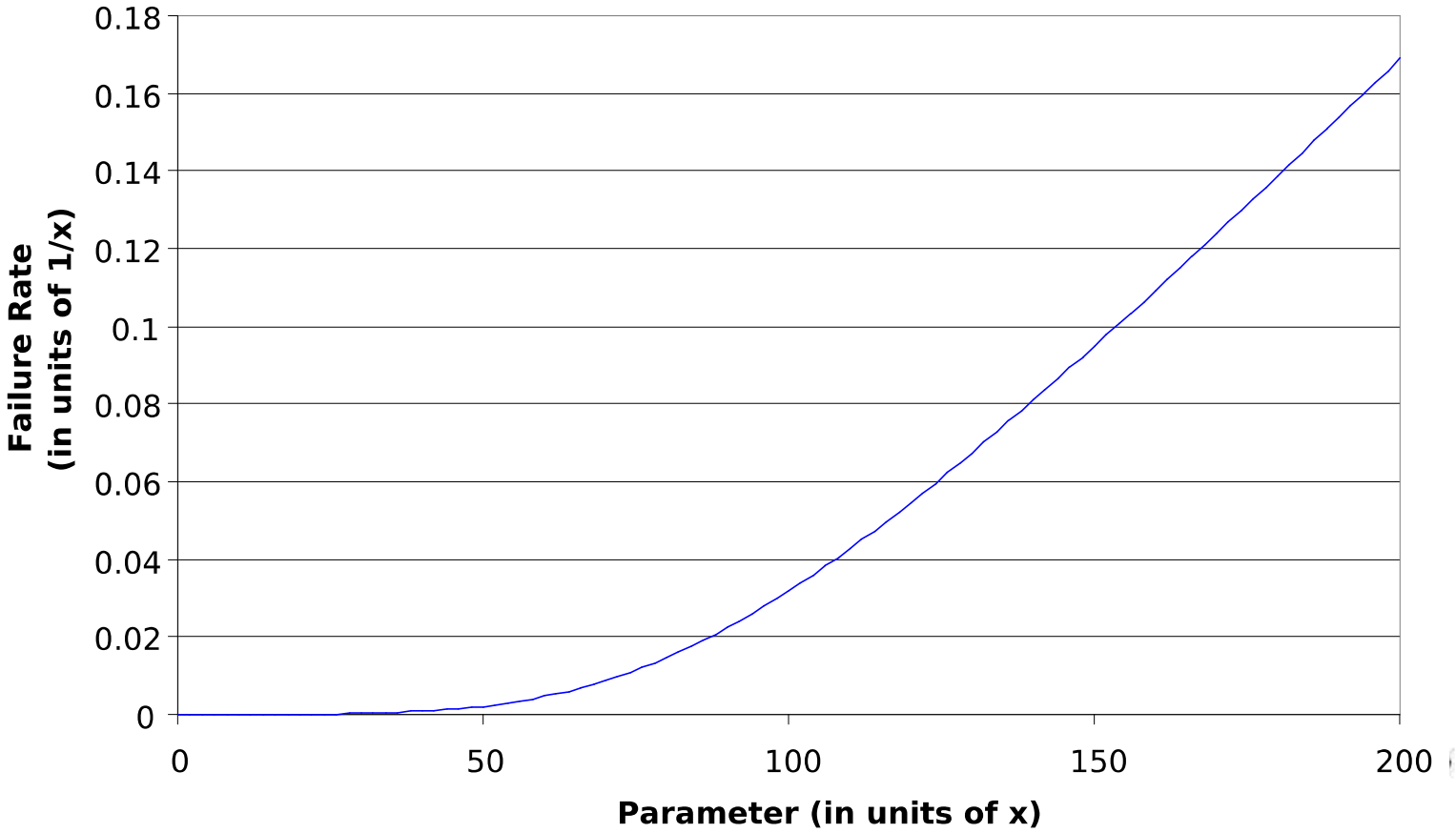




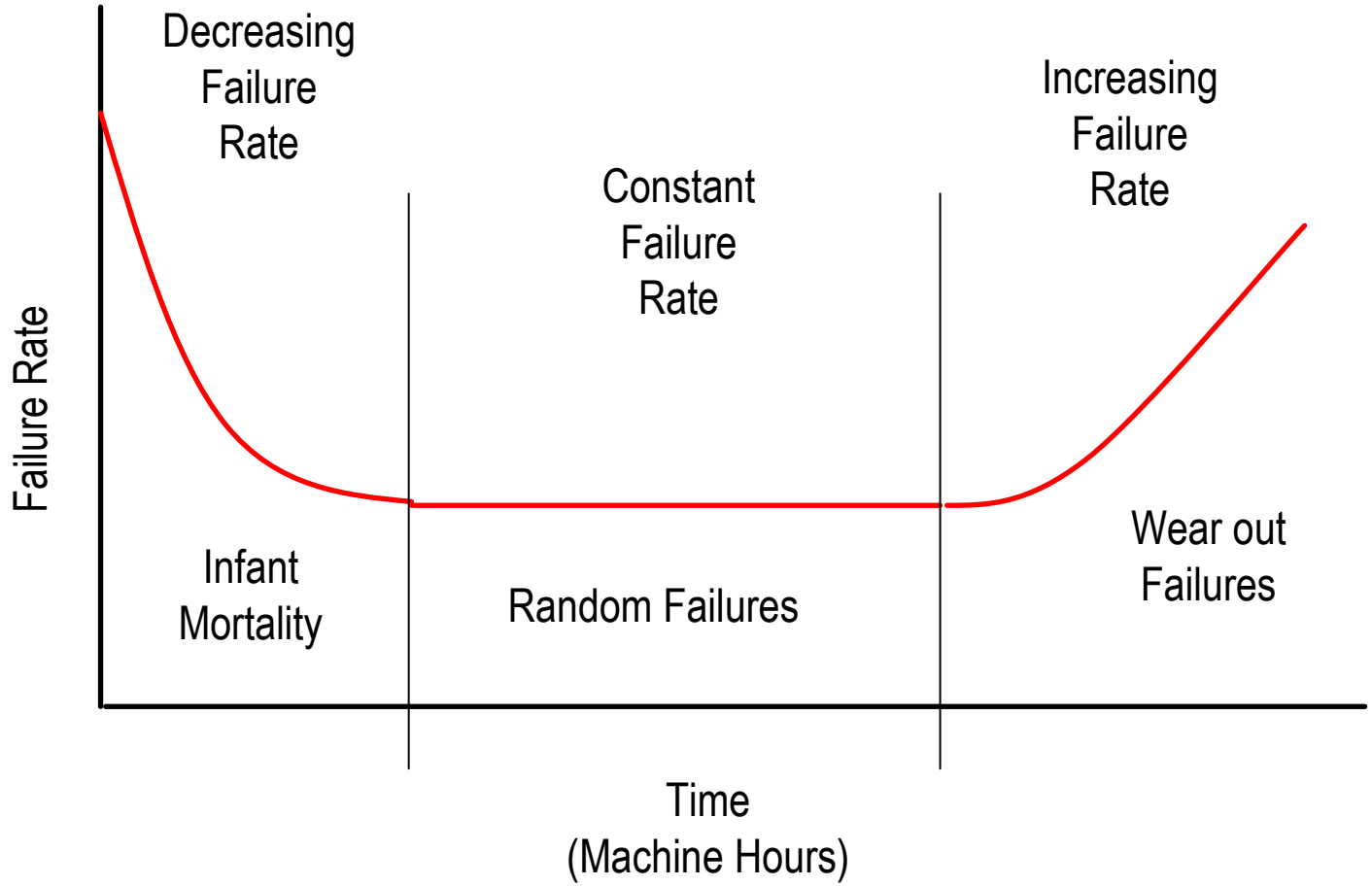
$$\text{Hazard} = \frac{\text{PDF}}{1-\text{CDF}}$$



Hazard Function



Bathtub Curve
(Hazard Function)



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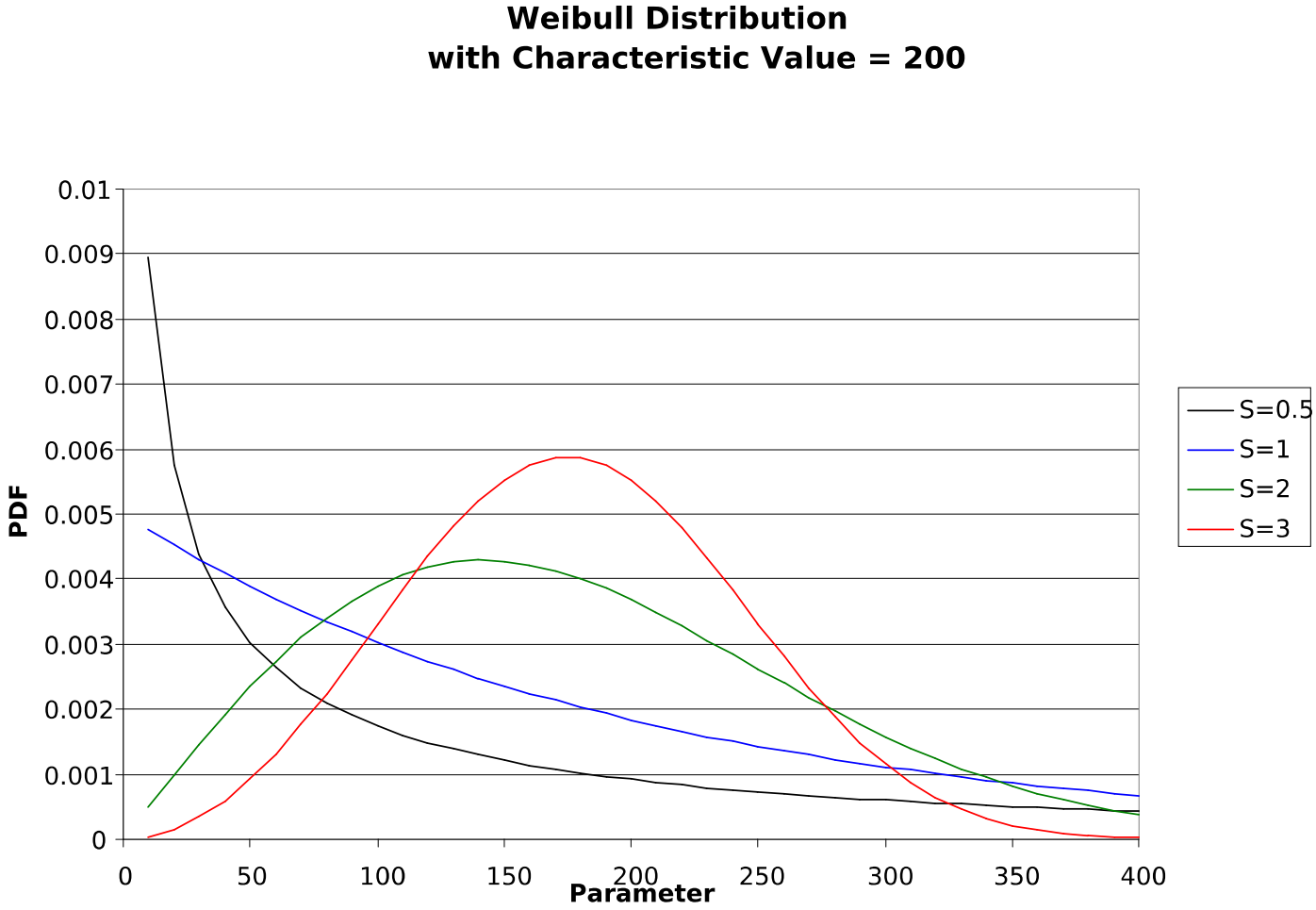

A Weibull Distribution is characterized by Slope and Characteristic Value

Weibull Distribution can be used to fit a wide variety of data.

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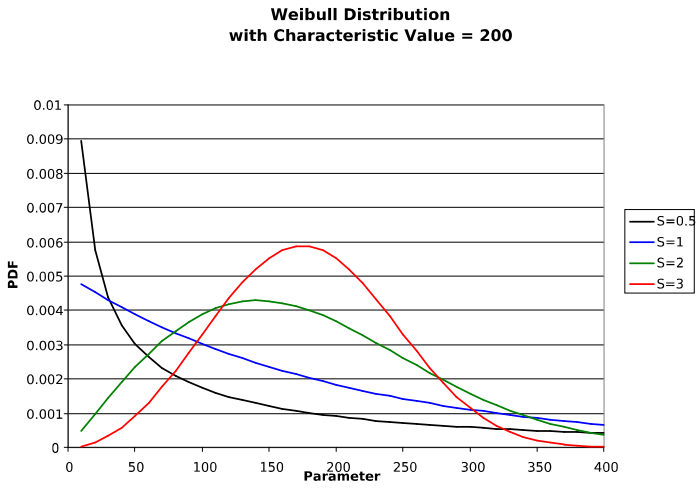
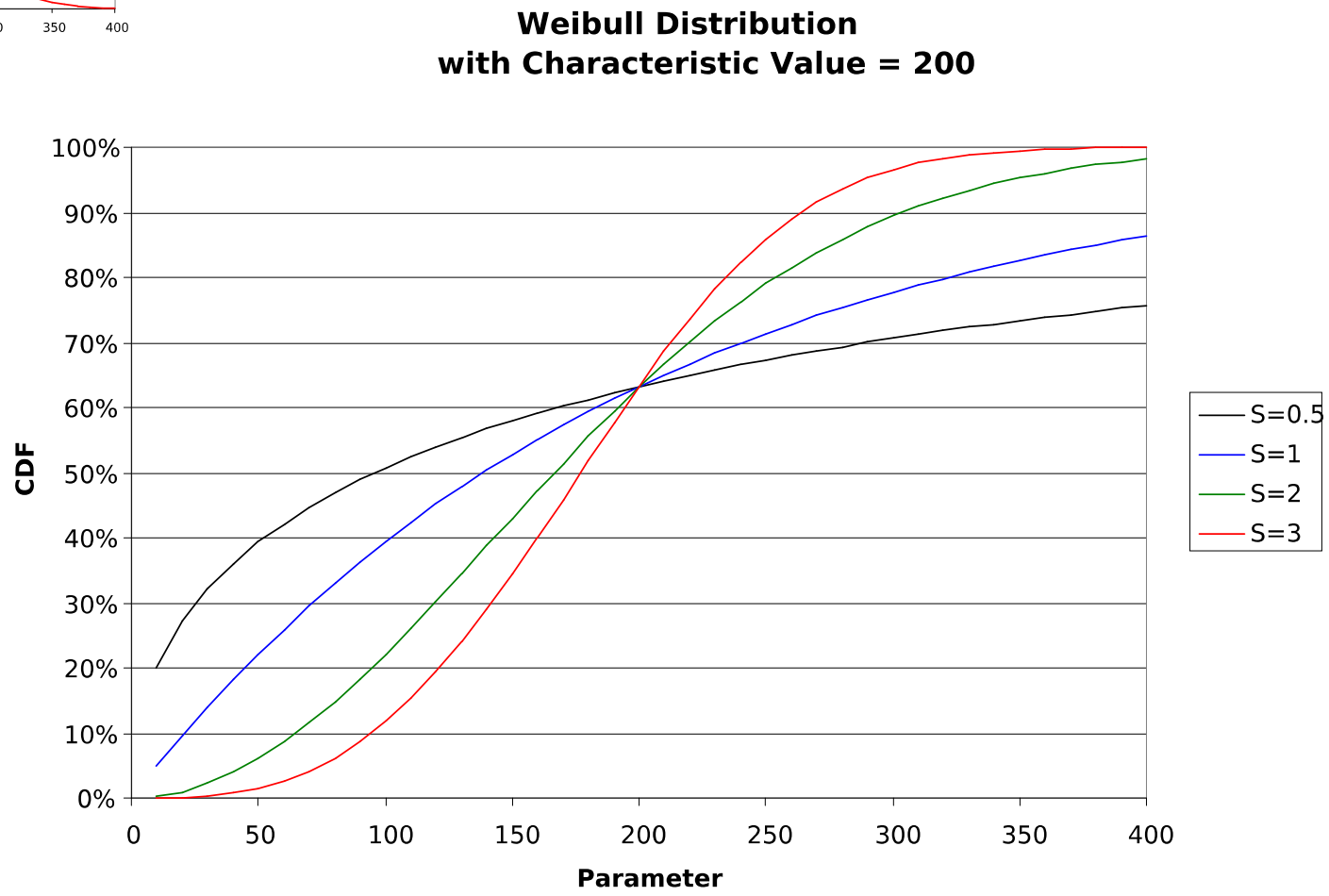
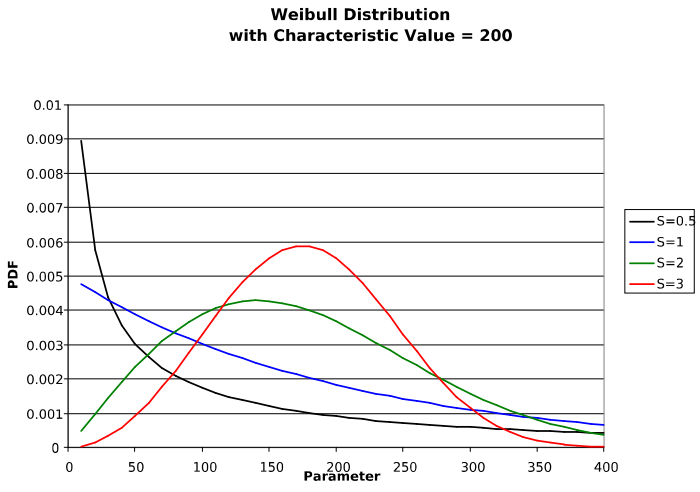
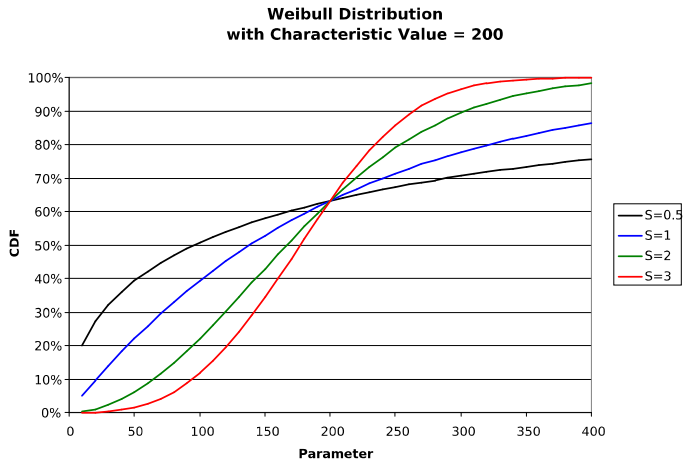
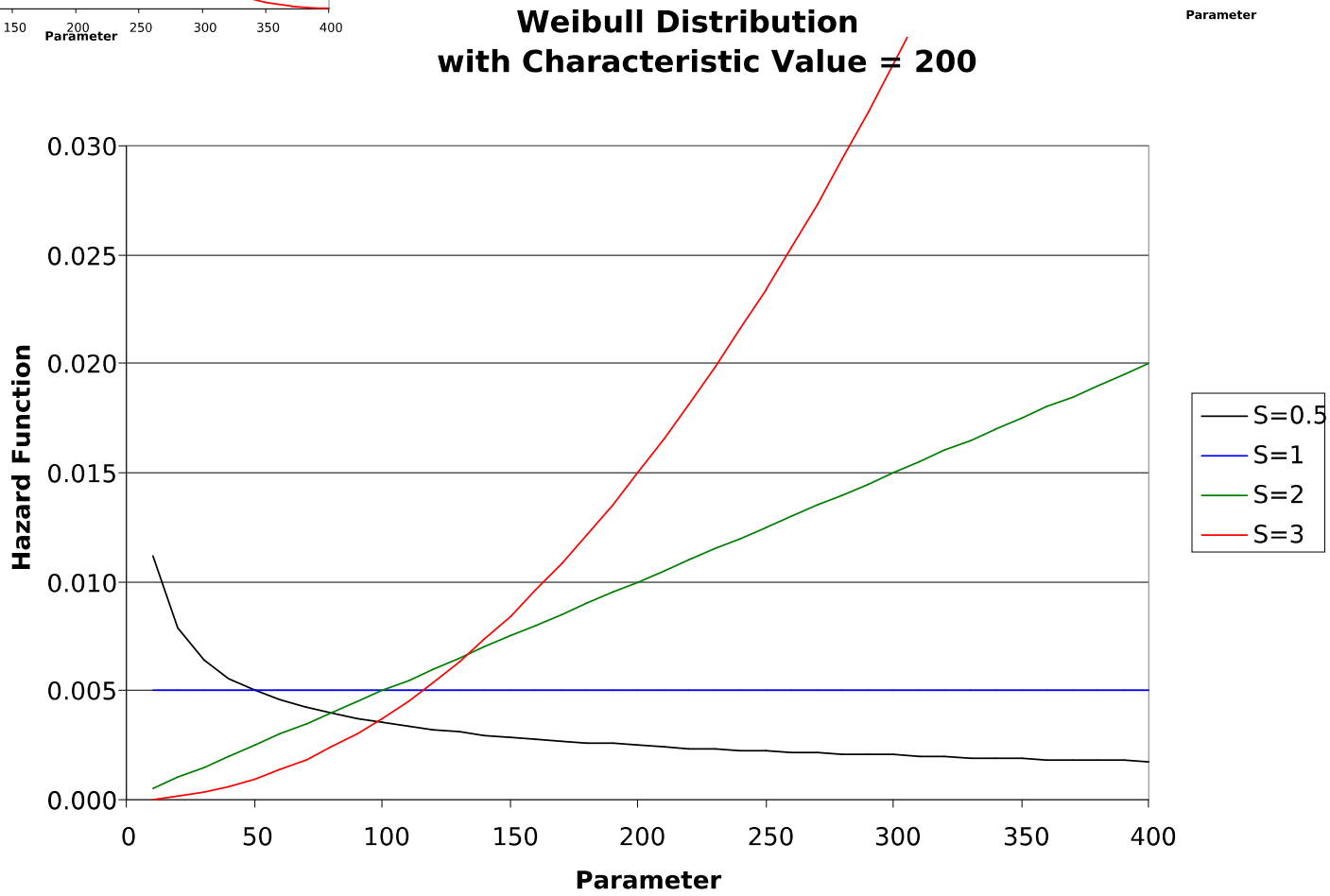


The Characteristic Values is where
The CDF = 63.2%

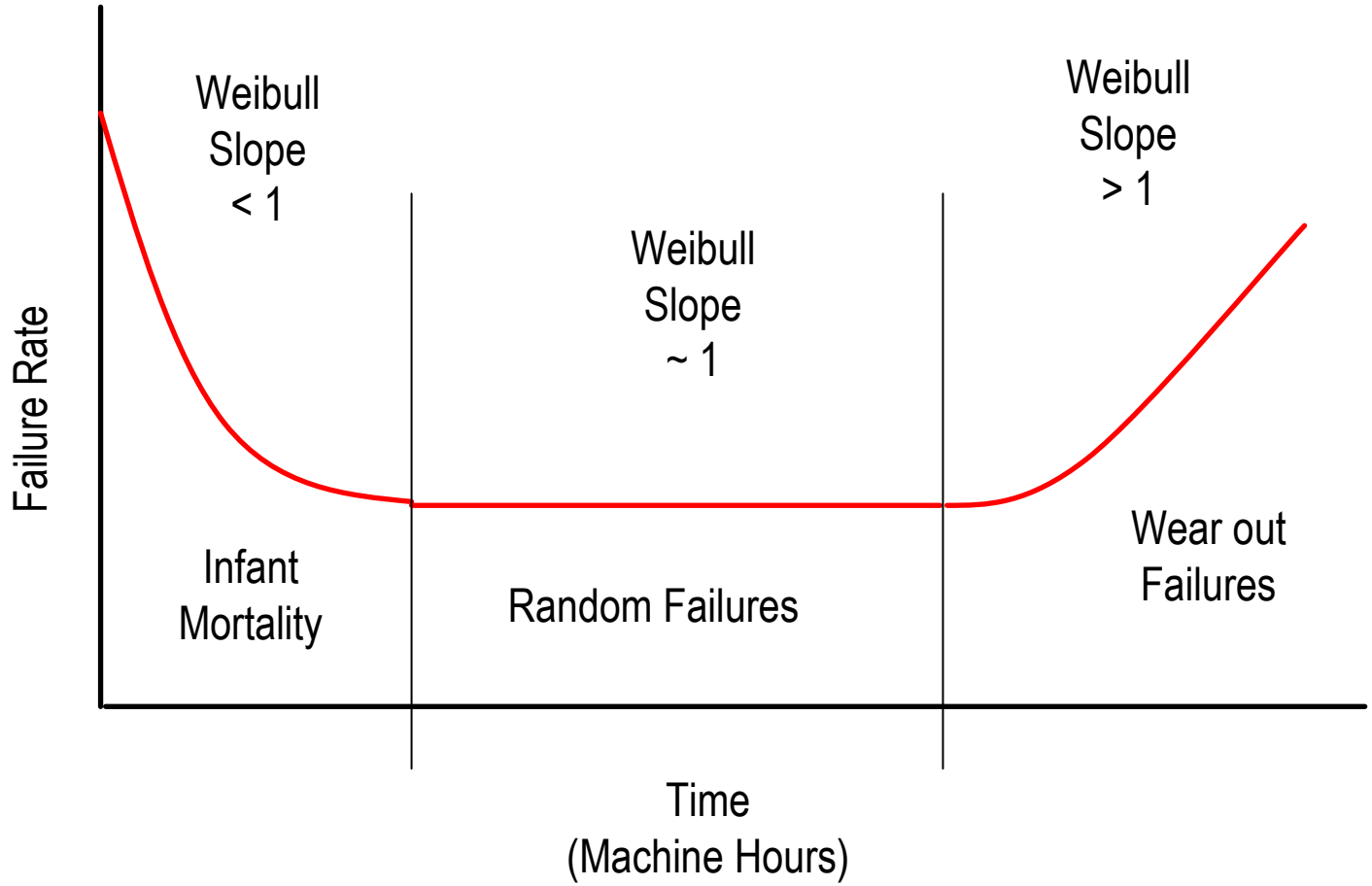



Weibull Distribution
with Characteristic Value = 200
PDF
Parameter
S=0.5
S=1
S=2
S=3


Weibull Distribution
with Characteristic Value = 200
CDF
Parameter
S=0.5
S=1
S=2
S=3


Weibull Distribution
with Characteristic Value = 200
Hazard Function
Parameter
S=0.5
S=1
S=2
S=3

Bathtub Curve
(Hazard Function)



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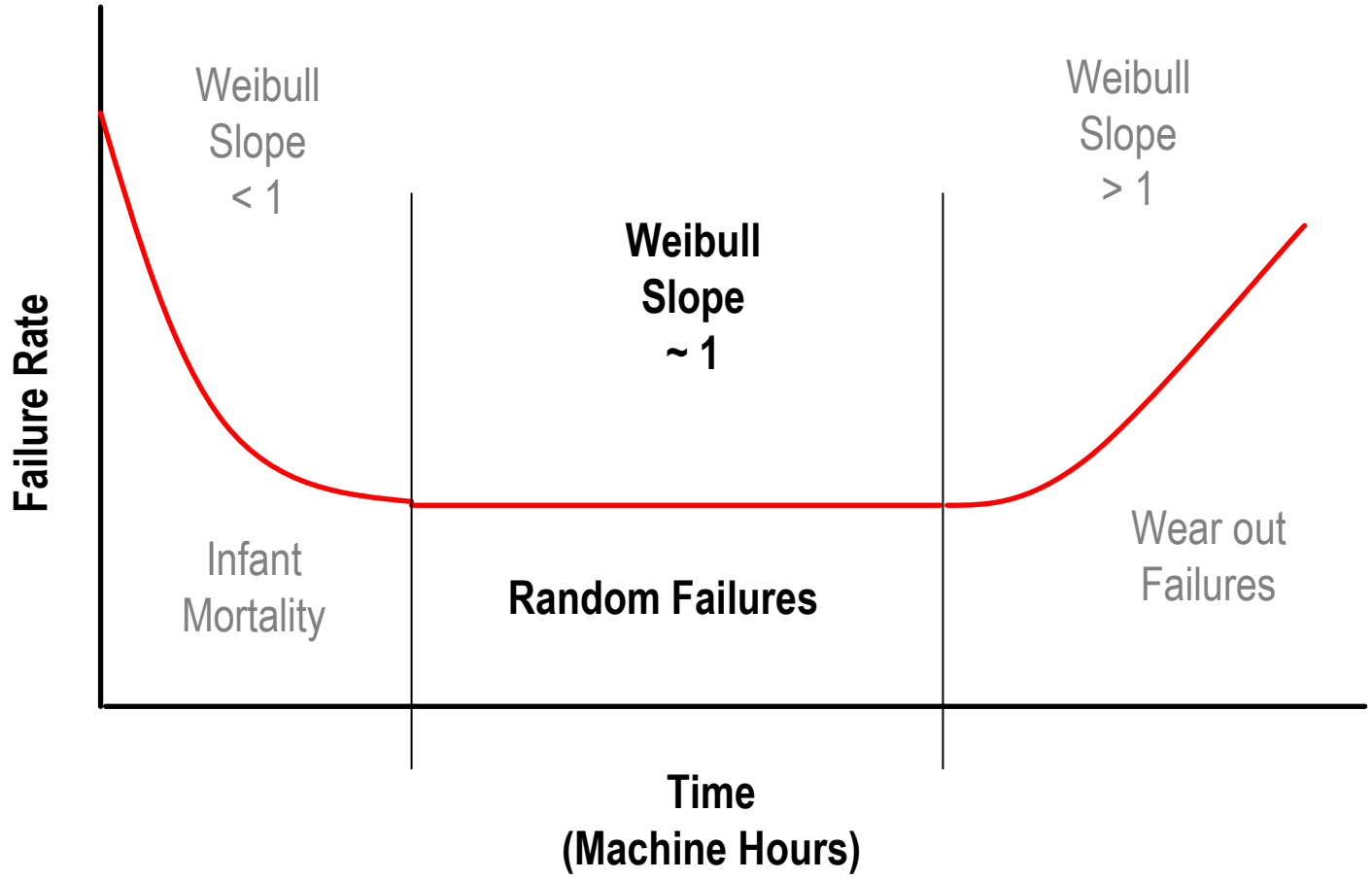
Bathtub Curve
(Hazard Function)



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Probabilistic Simulation

- 80 Hours of Measured Field Loads
- Statistical Distribution of Weld Fatigue Strength
- 10% COV on Geometry

- Randomly Generated 10,000 combinations
- Calculated the fatigue life of each combination
- Weibull Analysis of the resulting lives

- **Slope = 1.**

- Clearly a fatigue calculation but the failure rate was not dependent on time.

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Correct Way to View

- Probability of Failure = f(3 things)
  - **How it is made**
  - **How it is used**
  - **How long it is used**.

- Weibull analysis only looks at **How long it is used**.

- If the **variation** in **How it is made** and/or the variation in **How it is used** is very large, the effects of **How long it is used** are washed out.

- The Weibull slope is an indicator but it is not absolutely correct.

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Critical Success Factors in Engineering

1. Ask the right question.

2. Start with the basics.

3. Keep known things separate from assumptions.

4. Check your work.

5. Sometimes you have to say what no one wants to hear.
It should not leave them with a bad taste.

6. Never stop learning.

*

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